Chapter 8

Electromagnetic waves

8.1 Electromagnetism - a lightning review

The theory of electromagnetism revolves around the fields $\vec{E}$ and $\vec{B}$, defined through the Lorentz force law :

$$\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right) \tag{8.1}$$

In addition, we define auxiliary fields $\vec{D}$ and $\vec{H}$. Although today we consider $\vec{E}$ and $\vec{B}$ as the fundamental ones, and $\vec{D}$ and $\vec{H}$ as derived quantities, for historical reasons it is customary to consider $\vec{E}$ and $\vec{H}$ as fundamental fields.

Field	Definition	Constitutive Relations	Linear isotropic media	Vacuum
$\vec{D}$	$\epsilon_0\vec{E} + \vec{P}$	$\vec{D}\left[\vec{E},\vec{H}\right]$	$\epsilon\vec{E}$	$\epsilon_0\vec{E}$
$\vec{B}$	$\mu\vec{H} + \vec{M}$	$\vec{B}\left[\vec{E},\vec{H}\right]$	$\mu\vec{H}$	$\mu_0\vec{H}$

In the above, the fields $\vec{P}$ and $\vec{M}$ are the polarization and magnetization vector fields, respectively.

These fields are governed by the four Maxwell equations :

$$\begin{aligned} \nabla \cdot \vec{D} &= \rho && (8.2)\\ \nabla \cdot \vec{B} &= 0 && (8.3)\\ \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 && (8.4)\\ \nabla \times \vec{H} - \frac{\partial \vec{D}}{\partial t} &= \vec{j} && (8.5) \end{aligned}$$

where ρ and $\vec{j}$ are the charge and current densities, respectively! Taking the divergence of (8.5) and using (8.2) we land up with the equation of continuity

$$\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0. \tag{8.6}$$

The significance of this equation is clear from its integral version (which follows by integrating both sides over volume and using the Gauss divergence theorem)

$$\frac{dQ}{dt} = -\oint_{\Sigma} \vec{j} \cdot d\vec{s} \tag{8.7}$$

where Q is the total charge in the region bounded by the closed surface Σ. This implies that the total charge in a region decreases only by the charge flowing out of its surface, since $\vec{j} \cdot d\vec{s}$ is the charge crossing the surface area $d\vec{s}$ in unit time.

An important quantity for our discussion is the Poynting vector $\vec{S}$ defined by

$$\vec{S} = \vec{E} \times \vec{H} \tag{8.8}$$

Its significance can be seen by calculating its divergence

$$\begin{aligned} \nabla \cdot \vec{S} &= \nabla \cdot \left(\vec{E} \times \vec{H}\right) = \nabla \times \vec{E} \cdot \vec{H} - \vec{E} \cdot \nabla \times \vec{H} \\ &= -\frac{\partial \vec{B}}{\partial t} \cdot \vec{H} - \vec{E} \cdot \left(\vec{j} + \frac{\partial \vec{D}}{\partial t}\right) \end{aligned}$$

In the above we have used the vector calculus identity[1] $\nabla \cdot \left(\vec{A} \times \vec{B}\right) = \nabla \times \vec{A} \cdot \vec{B} - \vec{A} \cdot \nabla \times \vec{B}$. For linear isotropic media, we have

$$\nabla \cdot \vec{S} = -\vec{j} \cdot \vec{E} - \frac{\partial}{\partial t}\left(\frac{\epsilon}{2} E^2 + \frac{\mu}{2} H^2\right)$$

Since $\frac{\epsilon}{2} E^2 + \frac{\mu}{2} H^2$ is the elctromagnetic energy density w_{em}, this means that

$$\frac{\partial w_{em}}{\partial t} = -\vec{j} \cdot \vec{E} - \nabla \cdot \vec{S} \tag{8.9}$$

This means that the electromagnetic energy in a region can decrease in two ways - by means of Joule heating loss (the $-\vec{j} \cdot \vec{E}$ term) and by transport across the surface bounding the region (the $-\nabla \cdot \vec{S}$ term). So, we interpret the Poynting vector as the energy flowing through a surface normal to it per unit area per unit time.

Maxwell's equations also lead to the following continuity conditions for the fields at the interface of two media (in the absence of any surface charges or currents) :

[1] $\nabla \cdot \left(\vec{A} \times \vec{B}\right) = \partial_i \left(\epsilon_{ijk} A_j B_k\right) = \epsilon_{ijk} \left(\partial_i A_j\right) B_k + A_j \left(\epsilon_{ijk} \partial_i B_k\right) = \left(\nabla \times \vec{A}\right)_k B_k - A_j \left(\nabla \times \vec{B}\right)_j$

1. The tangential components of $\vec{E}$ and $\vec{H}$ are continuous across the interface
2. The normal components of $\vec{D}$ and $\vec{B}$ are continuous across the interface.

8.2 The wave equation in linear isotropic media

Consider a linear isotropic medium where there are no free charges. Here, Maxwell's equations become

$$\begin{aligned} \nabla \cdot \vec{E} &= 0 \quad (8.10) \\ \nabla \cdot \vec{B} &= 0 \quad (8.11) \\ \nabla \times \vec{E} + \frac{\partial \vec{B}}{\partial t} &= 0 \quad (8.12) \\ \nabla \times \vec{B} - \mu\epsilon \frac{\partial \vec{E}}{\partial t} &= 0 \quad (8.13) \end{aligned}$$

where we have used the constitutive relations $\vec{D} = \epsilon\vec{E}$ and $\vec{B} = \mu\vec{H}$ to express all the four equations in terms of $\vec{E}$ and $\vec{B}$. Note that (8.11) and (8.12) are independent of sources and medium - and are hence identical to our general equations (8.3) and (8.4).

We now take curl of both sides of Faraday's law, (8.12) to get

$$\nabla \times \left(\nabla \times \vec{E}\right) + \nabla \times \left(\frac{\partial \vec{B}}{\partial t}\right) = 0$$

Now we make use of the fact that the curl and $\frac{\partial}{\partial t}$ commute (which follows from the commutativity of mixed partial derivatives) and also the vector calculus identity[2] $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$ to get

$$\nabla\left(\nabla \cdot \vec{E}\right) - \nabla^2 \vec{E} + \frac{\partial}{\partial t}\left(\nabla \times \vec{B}\right) = 0.$$

Using (8.10) and (8.13) in the above, we get

$$\nabla^2 \vec{E} = \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad (8.14)$$

Propceeding in a similar fashion, but starting with (8.13) would give us

$$\nabla^2 \vec{B} = \mu\epsilon \frac{\partial^2 \vec{B}}{\partial t^2}. \quad (8.15)$$

[2] $\nabla \times \left(\nabla \times \vec{A}\right) = \epsilon_{ijk}\hat{e}_i\partial_j\left(\nabla \times \vec{A}\right)_k = \epsilon_{ijk}\hat{e}_i\partial_j\left(\epsilon_{klm}\partial_l A_m\right) = \epsilon_{ijk}\epsilon_{klm}\hat{e}_i\partial_j\partial_l A_m = \left(\delta_{ik}\delta_{jl} - \delta_{il}\delta_{kl}\right)\hat{e}_i\partial_j\partial_l A_m = \hat{e}_i\partial_i\partial_j A_j - \hat{e}_i\partial_j\partial_j A_i = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$

Thus both $\vec{E}$ and $\vec{B}$ obey the equation of a wave in three dimensions. So, in a charge free space in a linear isotropic dielectric the electric and magnetic fields propagate as a wave and the speed of this wave is

$$v = \frac{1}{\sqrt{\mu\epsilon}} \tag{8.16}$$

In particular, for the speed of this wave in vacuum, we have

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}}. \tag{8.17}$$

This comes out to be about 3×10^8 ms^{-1}, a value that matches the speed of light in vacuum (which is obviously much, much greater than the speed of anything else!). This coincidence in numerical value lead Maxwell to the hypothesis that light is an electromagnetic wave. Of course, this mere coincidence was not enough to convince anybody (even Maxwell himself) of the truth of this fact. As we will see, we can use Maxwell's equations to predict experimentally verifiable properties for electromagnetic waves - which turn out to match those of light! So succesful was Maxwell in predicting the properties of light in this way that it was soon accepted that science has finally uncovered the answer to the vexing question - "just what is waving in a light wave?"[3]

8.3 Plane progressive EM waves

As we have seen, a special solution of the three dimensional wave equation is the plane progressive wave. So, in a source-free linear isotropic dielectric, we have the solutions

$$\vec{E} = \vec{E}_0 e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)} \tag{8.18}$$
$$\vec{B} = \vec{B}_0 e^{i\left(\vec{k}\cdot\vec{r}-\omega t\right)} \tag{8.19}$$

where we have

$$\frac{\omega}{\left|\vec{k}\right|} = \frac{1}{\sqrt{\mu\epsilon}} = v.$$

Of course, we don't really mean that the electric or magnetic field is complex - the actual fields are real parts of the complex vectors.

We now proceed to find out what the Maxwell's equations have to say about the fields. For this we note that because our waves have

[3]Quite a lot is often made of the fact that historically quantum mechanics had started with a wave function (Schrödinger) even though the interpretation of just what was waving in that wave (Born) was missing for a long time. Note that in the case of light there was a gap of more than half a century between the discovery that light is a wave (Young) and the discovery that it is an elctromagnetic wave (Maxwell)!

an exponential space and time dependence - the derivative operators ∇ and $\frac{\partial}{\partial t}$ have very simple effects :

$$\nabla \equiv i\vec{k}, \qquad \frac{\partial}{\partial t} \equiv -i\omega \tag{8.20}$$

Using (8.20) in the four Maxwell's equations (8.10-8.13) leads to the relations

$$\begin{aligned} i\vec{k}\cdot\vec{E} &= 0, && \text{from Gauss law} \\ i\vec{k}\cdot\vec{B} &= 0, && \text{from Gauss law for magnetism} \\ i\vec{k}\times\vec{E} - i\omega\vec{B} &= 0, && \text{from Faraday's law} \\ i\vec{k}\times\vec{B} + \mu\epsilon i\omega\vec{E} &= 0, && \text{from Maxwell-Ampere law} \end{aligned}$$

The first two conditions tell us that electromagnetic waves are transverse - both the vectors $\vec{E}$ and $\vec{B}$ are normal to the direction of propagation $\vec{k}$. The third condition $\vec{B} = \omega^{-1}\vec{k}\times\vec{E}$ actually says that both of these vectors are actually normal to each other as well. Indeed, the three vectors $\vec{k}$, $\vec{E}$ and $\vec{B}$, taken in order, form a right handed triplet of mutually orthogonal vectors. Again, putting the last two of the equations together we get

$$\vec{E} = -\frac{\vec{k}\times\vec{B}}{\mu\epsilon\omega} = -\frac{\vec{k}\times\left(\vec{k}\times\vec{E}\right)}{\mu\epsilon\omega^2} = \frac{\left|\vec{k}\right|^2}{\mu\epsilon\omega^2}\vec{E}$$

where we have used $\vec{k}\cdot\vec{E} = 0$ in the last part. For this to be consistent, we must have

$$\frac{\omega^2}{\left|\vec{k}\right|^2} = \frac{1}{\mu\epsilon} = v^2$$

Thus, the plane progressive waves (8.18) and (8.19) denote solutions to the Maxwell's equations only if

$$\omega = vk$$

where $k = \left|\vec{k}\right|$ is the wavenumber of the wave. With this condition in place, the four conditions above can be reduced to the two statements

$$\vec{k}\cdot\vec{E} = 0 \tag{8.21}$$

$$\vec{B} = \frac{\vec{k}\times\vec{E}}{\omega} = \frac{\hat{n}\times\vec{E}}{v} \tag{8.22}$$

where $\hat{n} \equiv \frac{\vec{k}}{k}$ is the unit vector in the direction of propagation of the wave.

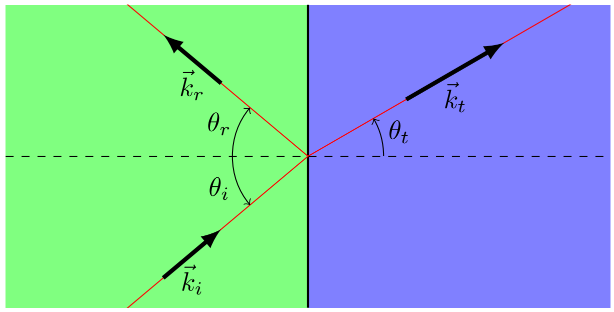

Figure 8.1: Geometry of reflection and transmission at a planar interface

Let us now find the Poynting vector corresponding to such a plane progressive wave. This is

$$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu}\vec{E} \times \vec{B} = \frac{1}{\mu v}\,\vec{E} \times \left(\hat{n} \times \vec{E}\right)$$

where we have used (8.22). Using the fact that $\hat{n} \cdot \vec{E} = 0$ we arrive at

$$\vec{S} = \frac{E^2}{\mu v}\,\hat{n} \tag{8.23}$$

Thus the amount of energy a plane progressive electromagnetic wave carries per unit area per unit time is propotional to E^2 , the constant of proportionality being $\frac{1}{\mu v}$. In addition, this also shows that the energy flows in the same direction as that of wave propagation (although this sounds inevitable - it actually is not the case when light travels through an anisotropic medium!).

8.4 Reflection and transmission of electromagnetic waves at a planar interface

Let us now turn to consider what will happen if a plane progressive electromagnetic wave travelling through a linear isotropic medium where its speed is v_1 were to be incident at an interface with another such medium, where its speed is v_2. For simplicity, we consider both media to be semi-infinite and their interface to be a plane. In the following, the incident, the reflected and the transmitted waves are denoted by the indices i, r and t respectively.

What will happen here is determined by the continuity conditions

1. The tangential components of $\vec{E}$ and $\vec{H}$ are continuous across the interface

2. The normal components of $\vec{D}$ and $\vec{B}$ are continuous across the interface.

The fact that these conditions must be obeyed at all times and at all points on the interface has immediate consequences.

8.4.1 Laws of reflection and transmission (refraction)

Since the time dependence of the incident, reflected and transmitted fields are governed by the factors $e^{-i\omega_i t}$, $e^{-i\omega_r t}$ and $e^{-i\omega_t t}$, respectively, we must have

$$\omega_i = \omega_r = \omega_t = \omega\,(\text{say})\,.$$

Thus we have an explanation for the very important fact that frequency does not change on reflection or transmission. Again, the fact that the continuity conditions must work all over the interface tells us that $\vec{k}_i \cdot \vec{r}$, $\vec{k}_r \cdot \vec{r}$ and $\vec{k}_t \cdot \vec{r}$ must change by the same amount as we move on the interface. This means that for two points $\vec{r}_1$ and $\vec{r}_2$ on the interface, we must have

$$\vec{k}_i \cdot \vec{r}_1 - \vec{k}_i \cdot \vec{r}_2 = \vec{k}_r \cdot \vec{r}_1 - \vec{k}_r \cdot \vec{r}_2 = \vec{k}_t \cdot \vec{r}_1 - \vec{k}_t \cdot \vec{r}_2$$

Since $\vec{r}_1$ and $\vec{r}_2$ are arbitrary points *on* the interface, we must have

$$\vec{k}_i \cdot \vec{\rho} = \vec{k}_r \cdot \vec{\rho} = \vec{k}_t \cdot \vec{\rho}$$

where $\vec{\rho}$ is an arbitrary vector on the interface. This means that the vectors $\vec{k}_i - \vec{k}_r$ and $\vec{k}_i - \vec{k}_t$ are normal to the interface. Thus both $\vec{k}_r$ and $\vec{k}_t$ are in the plane of incidence - which is the plane defined by $\vec{k}_i$ - the direction of incidence and $\hat{N}$ - the normal to the interface. Again, we must have

$$\hat{N} \times \vec{k_i} = \hat{N} \times \vec{k_r} = \hat{N} \times \vec{k_t}$$

Taking magnitudes of both sides give us

$$\left|\vec{k}_i\right| \sin\theta_i = \left|\vec{k}_r\right| \sin\theta_r = \left|\vec{k}_t\right| \sin\theta_t$$

Since the frequency is the same for all three waves we have

$$\sin\theta_i = \sin\theta_r = \frac{v_1}{v_2} \sin\theta_t$$

The first equality above gives us the law of reflection, $\theta_i = \theta_r$ - while the second one gives us Snell's law :

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = n \tag{8.24}$$

where $n \equiv \frac{v_1}{v_2}$ is the refractive index of the second medium with respect to the first.

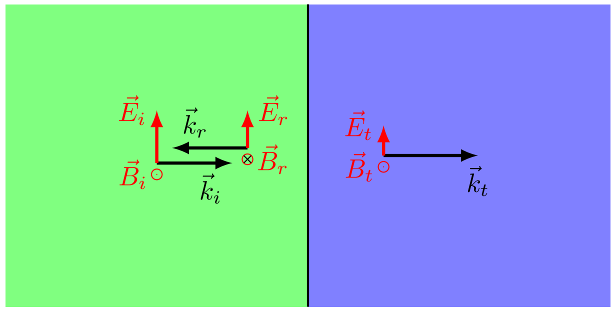

Figure 8.2: Reflection and transmission for normal incidence

8.4.2 Normal incidence

We next turn our attention to the question of how much of the incident energy is reflected and how much is transmitted. As a warm-up exercise, we will start with the easier case of normal incidence, $\theta_i = 0$. Let the incident electric field $\vec{E}_i$ be in the plane of the paper at a given instant of time as shown in figure (8.2) . In this case, the field $\vec{B}_i$ will be pointing out of the plane of the paper. A reflection in the plane of the paper will leave the incident fields the same (note that $\vec{B}_i$, being a pseudo-vector stays the same despite being perpendicular to the plane of the reflection), and will also not affect the two media. This means that the resulting reflected and transmitted electromagnetic fields will also stay the same under this reflection. Thus both $\vec{E}_r$ and $\vec{E}_t$ must be in the plane (and being perpendiculer to $\vec{k}_r$ and $\vec{k}_t$, respectively, must be in the directions shown), and $\vec{B}_r$ and $\vec{B}_t$ out of the plane (for the directions of $\vec{E}_r$ and $\vec{E}_t$ shown, $\vec{B}_t$ must be out of the plane and $\vec{B}_r$ into the plane). For this particular geometry, the normal components of each of the fields is zero - and thus the continuity conditions for D_n and B_n are trivially satisfied. The continuity conditions for $\vec{E}_t$ and $\vec{H}_t$ gives us the following :

$$\begin{aligned} E_i + E_r &= E_t \\ \frac{1}{\mu_1 v_1}(E_i - E_r) &= \frac{1}{\mu_2 v_2} E_t \end{aligned}$$

These can be easily solved to get

$$\begin{aligned} \frac{E_t}{E_i} &= \frac{2}{1 + \frac{\mu_1}{\mu_2} n} \to \frac{2}{1+n} \\ \frac{E_r}{E_i} &= \frac{1 - \frac{\mu_1}{\mu_2} n}{1 + \frac{\mu_1}{\mu_2} n} \to \frac{1-n}{1+n} \end{aligned}$$

where the last part in each expression is the approximation for non-magnetic materials for which $\mu_1 \approx \mu_2 \approx \mu_0$ (Note that most magnetic substances are opaque to light anyway - so this is usually a good approximation to make). To find the amount of incident energy that is reflected and transmitted we must use the Poynting vector :

$$R \equiv \frac{\left|\vec{S}_r\right|}{\left|\vec{S}_i\right|} = \frac{\left|\vec{E}_r\right|^2}{\left|\vec{E}_i\right|^2} = \left(\frac{1-\frac{\mu_1}{\mu_2}n}{1+\frac{\mu_1}{\mu_2}n}\right)^2 \to \left(\frac{1-n}{1+n}\right)^2 \tag{8.25}$$

$$T \equiv \frac{\left|\vec{S}_t\right|}{\left|\vec{S}_i\right|} = \frac{\mu_1 v_1}{\mu_2 v_2}\frac{\left|\vec{E}_t\right|^2}{\left|\vec{E}_i\right|^2} = \frac{4\frac{\mu_1}{\mu_2}n}{\left(1+\frac{\mu_1}{\mu_2}n\right)^2} \to \frac{4n}{(1+n)^2} \tag{8.26}$$

As expected from conservation of energy, we have

$$R + T = 1. \tag{8.27}$$

Equations (8.25) and (8.26) are called Fresnel's equations for the special case of normal incidence. We now turn to the more general case of oblique incidence.

8.4.3 Fresnel's equations for oblique incidence

In the case of oblique incidence, the incident wave vector $\vec{k}_i$ and the normal to the interface $\hat{N}$ together define a unique plane of incidence - which, as we have already seen, also contains the wavevectors $\vec{k}_r$ and $\vec{k}_t$. Reflecting the system in this plane does leave the wavevectors and the media intact. However, since the electric and magnetic fields in this case are not necessarily in and out of the plane of incidence in this case, we no longer have the symmetry that we used while arguing about normal incidence. However in this case it is easy to see that if we consider the electric field to be in the plane of incidence (and then the corresponding magnetic field must be perpendicular to it) then we do have the symmetry and hence the resulting fields $\vec{E}_r$ and $\vec{E}_t$ must be in the plane of incidence as before. A similar argument shows easily that if the incident $\vec{E}_i$ is perpendicular to the plane of incidence, then so must be $\vec{E}_r$ and $\vec{E}_t$. In this case, it makes sense to consider these two cases separately. The case of the more general direction of $\vec{E}_i$ can be easily treated by superposition.

8.4.3.1 Electric fields in the plane of incidence

It is easy to see that the continuity conditions for $\vec{E}_t$ and $\vec{H}_t$ gives us, in this case, the two equations

$$\begin{aligned} E_i \cos\theta_i + E_r \cos\theta_r &= E_t \cos\theta_t \\ \frac{1}{\mu_1 v_1}\left(E_i - E_r\right) &= \frac{1}{\mu_2 v_2} E_t \end{aligned}$$

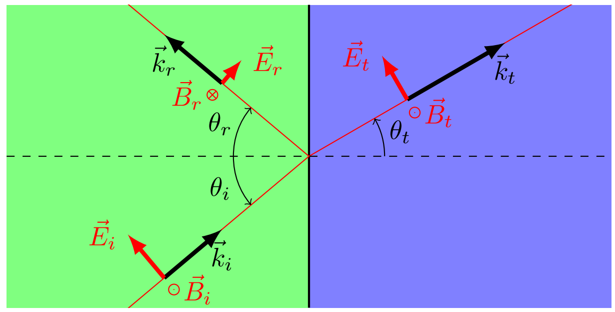

Figure 8.3: Reflection and transmission at a planar interface for the incident electric field in the plane of incidence

which can be rewritten in the form

$$\begin{aligned} E_i + E_r &= \frac{\cos\theta_t}{\cos\theta_i} E_t \\ E_i - E_r &= \frac{\mu_1}{\mu_2} n\, E_t \end{aligned}$$

which are solved easily to yield

$$\begin{aligned} \frac{E_r}{E_i} &= \frac{\frac{\cos\theta_t}{\cos\theta_i} - \frac{\mu_1}{\mu_2} n}{\frac{\cos\theta_t}{\cos\theta_i} + \frac{\mu_1}{\mu_2} n} \rightarrow \frac{\frac{\cos\theta_t}{\cos\theta_i} - n}{\frac{\cos\theta_t}{\cos\theta_i} + n} \\ \frac{E_t}{E_i} &= \frac{2}{\frac{\cos\theta_t}{\cos\theta_i} + \frac{\mu_1}{\mu_2} n} \rightarrow \frac{2}{\frac{\cos\theta_t}{\cos\theta_i} + n} \end{aligned}$$

To find the reflection and transmission coefficients in this case we must consider the energy carried away in a direction normal to the interface by the reflected and transmitted waves. Thus

$$R \equiv \frac{\left|\vec{S}_r \cdot \hat{N}\right|}{\left|\vec{S}_i \cdot \hat{N}\right|} = \frac{\left|\vec{E}_r\right|^2}{\left|\vec{E}_i\right|^2} = \left(\frac{\frac{\cos\theta_t}{\cos\theta_i} - \frac{\mu_1}{\mu_2} n}{\frac{\cos\theta_t}{\cos\theta_i} + \frac{\mu_1}{\mu_2} n}\right)^2 \rightarrow \left(\frac{\frac{\cos\theta_t}{\cos\theta_i} - n}{\frac{\cos\theta_t}{\cos\theta_i} + n}\right)^2 \tag{8.28}$$

$$T \equiv \frac{\left|\vec{S}_t \cdot \hat{N}\right|}{\left|\vec{S}_i \cdot \hat{N}\right|} = \frac{\frac{1}{\mu_2 v_2}\left|\vec{E}_t\right|^2 \cos\theta_t}{\frac{1}{\mu_1 v_1}\left|\vec{E}_i\right|^2 \cos\theta_i} = \frac{4\frac{\mu_1}{\mu_2} n \frac{\cos\theta_t}{\cos\theta_i}}{\left(\frac{\cos\theta_t}{\cos\theta_i} + \frac{\mu_1}{\mu_2} n\right)^2} \rightarrow \frac{4n\frac{\cos\theta_t}{\cos\theta_i}}{\left(\frac{\cos\theta_t}{\cos\theta_i} + n\right)^2} \tag{8.29}$$

As you can easily check, we get $R + T = 1$ as expected.

If $n > 1$ it is well known that the transmitted beam bends towards the normal, *i.e.* $\theta_t \leq \theta_i$, so that $\frac{\cos\theta_t}{\cos\theta_i} \geq 1$. Thus it is possible for R to vanish for some θ_i. The same holds for $n < 1$. It is easy to see that this happens for

$$\frac{\cos\theta_t}{\cos\theta_i} = n = \frac{\sin\theta_i}{\sin\theta_t}$$

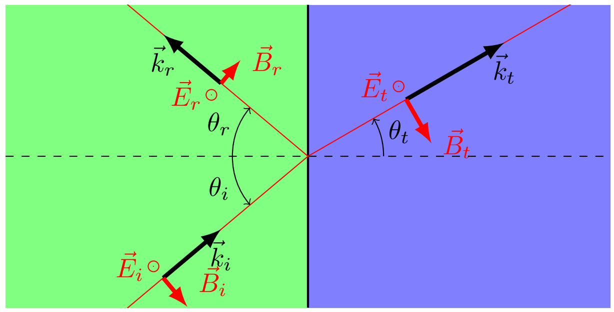

Figure 8.4: Reflection and transmission at a planar interface for the incident electric field out of the plane of incidence

so that $\sin(2\theta_i) = \sin(2\theta_t)$. Since $\theta_t \neq \theta_i$ ($\theta_t = \theta_i$ is possible only for normal incidence, in which case $R = \left(\frac{1-n}{1+n}\right)^2 \neq 0$), we must have

$$2\theta_t = \pi - 2\theta_i$$

or

$$\theta_t + \theta_r = \frac{\pi}{2}$$

which means geometrically that the reflected and transmitted beams are perpendicular to each other. The angle of incidence is then easily seen to satisfy

$$\tan\theta_i = n$$

This angle of incidence is called the Brewster angle. At this angle of incidence, the reflected beam is completely polarized with the electric field perpendicular to the plane of incidence (as you will see soon, this component does not vanish!).

8.4.3.2 Electric fields normal to the plane of incidence

In this case, reflection in the plane of incidence reverses both the incident electric field and the incident magnetic field (which is in the plane of incidence in this case). Thus, the resulting reflected and transmitted fields must be reversed on reflection as well - which means that both $\vec{E}_t$ and $\vec{E}_r$ must be normal to the plane of incidence as well.

Continuity of $\vec{E}_t$ and $\vec{H}_t$ leads in this case to the equations

$$\begin{aligned} E_i + E_r &= E_t \\ \frac{\cos\theta_i}{\mu_1 v_1}(E_i - E_r) &= \frac{\cos\theta_t}{\mu_2 v_2} E_t \end{aligned}$$

which leads to the solution

$$\begin{aligned}
\frac{E_r}{E_i} &= \frac{1-\frac{\mu_1}{\mu_2}n\frac{\cos\theta_t}{\cos\theta_i}}{1+\frac{\mu_1}{\mu_2}n\frac{\cos\theta_t}{\cos\theta_i}} \to \frac{1-n\frac{\cos\theta_t}{\cos\theta_i}}{1+n\frac{\cos\theta_t}{\cos\theta_i}} \\
\frac{E_t}{E_i} &= \frac{2}{1+\frac{\mu_1}{\mu_2}n\frac{\cos\theta_t}{\cos\theta_i}} \to \frac{2}{1+n\frac{\cos\theta_t}{\cos\theta_i}}
\end{aligned}$$

In this case, the reflection and transmission coefficients are

$$R = \left(\frac{1-\frac{\mu_1}{\mu_2}n\frac{\cos\theta_t}{\cos\theta_i}}{1+\frac{\mu_1}{\mu_2}n\frac{\cos\theta_t}{\cos\theta_i}}\right)^2 \to \left(\frac{1-n\frac{\cos\theta_t}{\cos\theta_i}}{1+n\frac{\cos\theta_t}{\cos\theta_i}}\right)^2 \tag{8.30}$$

$$T = \frac{4\frac{\mu_1}{\mu_2}n\frac{\cos\theta_t}{\cos\theta_i}}{\left(1+\frac{\mu_1}{\mu_2}n\frac{\cos\theta_t}{\cos\theta_i}\right)^2} \to \frac{4n\frac{\cos\theta_t}{\cos\theta_i}}{\left(1+n\frac{\cos\theta_t}{\cos\theta_i}\right)^2} \tag{8.31}$$

which obey $R+T=1$, as always!